Logic Synthesis via Boolean Relations

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A. Bernasconi, V. Ciriani, G. Trucco, T. Villa,
Using Flexibility in P-Circuits by Boolean Relations,
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Outline

- Boolean relations
- Example: logic synthesis with critical signals
- Problem definition
- P-circuits
- Synthesis of P-circuits with Boolean relations
- Experimental results

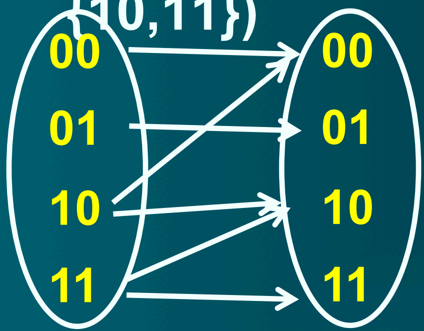
Incompletely specified Multioutput Boolean function

- **Incompletely specified *n*-input, *m*-output Boolean function:**

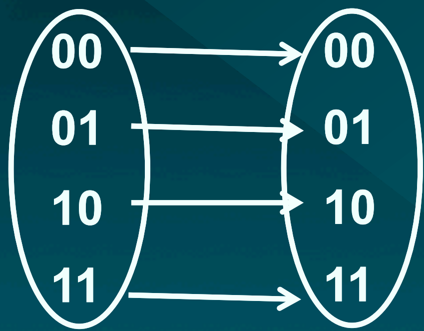
$$F: \{0,1\}^n \rightarrow \{0,1,-\}^m$$

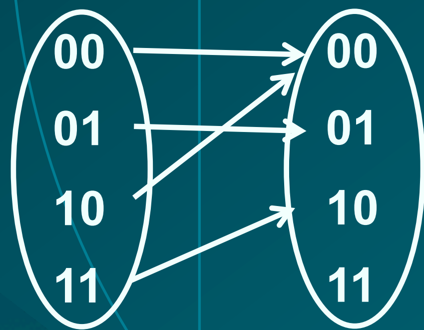
- **Example**

  F(00) = 00, F(01) = 01, F(10) = - 0 (i.e., F(10) = {00,10}), F(11) = 1- (i.e., F(11) = {10,11})



Incompletely specified Multioutput Boolean function



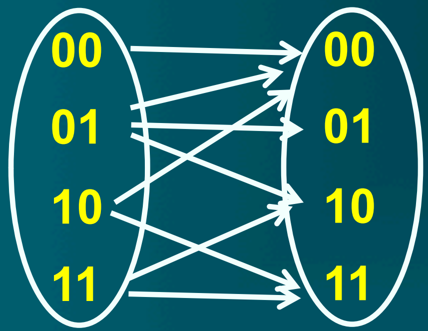


Some covering functions

Boolean Relations

- Boolean relations are a generalization of incompletely specified logic functions
- *A **Boolean relation** R: $\{0,1\}^n \rightarrow \{0,1\}^m$* is a one-to-many multi-output Boolean **mapping**
- Example

R(00) = {00}
R(01) = {00, 01, 10}
R(10) = {00,11}
R(11) = {10,11} = 1-



Boolean relation

Covering functions (compatible functions)

Boolean Relations

- The set of multi-output functions compatible with a Boolean relation R is defined as

$$F(R) = \{ f \mid f \subseteq R \text{ and } f \text{ is a function} \}.$$

- The *solution* of a Boolean relation R is a multi-output Boolean function $f \in F(R)$

- The function f is an *optimal solution* of R according to a given cost function *c*, if

$$\forall f' \in F(R),\ c(f) \leq c(f')$$

Example of Synthesis via Boolean Relations

- **Scenario:**
  - **Logic synthesis in presence of critical signals that should be moved toward the output**
- **Application fields:**
  - **for decreasing power consumption:**
    - **signals with high switching activity**
  - **for decreasing circuit delay:**
    - **signals with high delay**

Problem

Restructure (or synthesize) a circuit in order to move critical signals near to the output (decreasing the cone of influence):

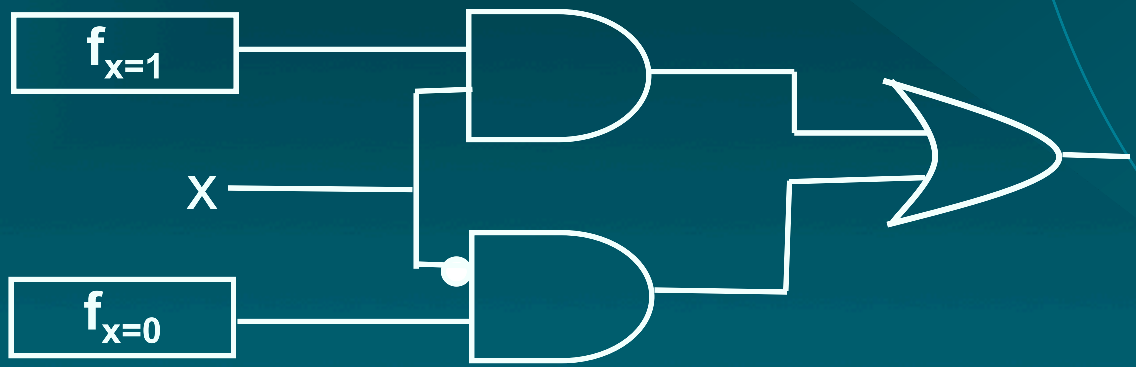
- minimizing the circuit area
- keeping the number of levels bounded
- performing an efficient minimization

Simple solution: Shannon

- Shannon decomposition
- x is the **critical signal**

$$f = x\, f_{x=1} + \overline{x}\, f_{x=0}$$

- $f_{x=0}$ and $f_{x=1}$ **do not depend** on x
- x is near to the output



Problem of Shannon approach

Let $x = x_1$

$x_1 x_2$ \ $x_3 x_4$	00	01	11	10
00	0	1	1	0
01	0	0	1	0
11	1	1	1	1
10	0	0	1	0

$x_1 = 0$

x_2 \ $x_3 x_4$	00	01	11	10
0	0	1	1	0
1	0	0	1	0

two cubes

$x_1 = 1$

x_2 \ $x_3 x_4$	00	01	11	10
0	0	0	1	0
1	1	1	1	1

It is not a compact representation

The idea

- try not to split the cubes
- let the critical signal x near to the output
- idea:

  the crossing cubes that do not depend on x are not projected

- problem: how to identify the points that may form crossing cubes that do not depend on x?

They are in the intersection: $I = f_{x=0} \cap f_{x \neq 0}$

Example

We can remove the points of the intersection

$x_1 x_2$ \ $x_3 x_4$	00	01	11	10
00	0	1	1	0
01	0	0	1	0
11	1	1	1	1
10	0	0	1	0

$x = x_1$

$x_1 = 0$

x_2 \ $x_3 x_4$	00	01	11	10
0	0	1	1	0
1	0	0	1	0

$x_1 = 1$

x_2 \ $x_3 x_4$	00	01	11	10
0	0	0	1	0
1	1	1	1	1

Intersection

x_2 \ $x_3 x_4$	00	01	11	10
0	0	0	1	0
1	0	0	1	0

Example

We can remove the points of the intersection

$x_1 x_2$ \ $x_3 x_4$	00	01	11	10
00	0	1	1	0
01	0	0	1	0
11	1	1	1	1
10	0	0	1	0

But some cubes could be split!

$x_1 = 0$

x_2 \ $x_3 x_4$	00	01	11	10
0	0	1	0	0
1	0	0	0	0

$x_1 = 1$

x_2 \ $x_3 x_4$	00	01	11	10
0	0	0	0	0
1	1	1	0	1

Intersection

x_2 \ $x_3 x_4$	00	01	11	10
0	0	0	1	0
1	0	0	1	0

Example

We insert don't cares instead to the points of the intersection

$x_1 x_2$ \ $x_3 x_4$	00	01	11	10
00	0	1	1	0
01	0	0	1	0
11	1	1	1	1
10	0	0	1	0

$x_1 = 0$

x_2 \ $x_3 x_4$	00	01	11	10
0	0	1	-	0
1	0	0	-	0

$x_1 = 1$

x_2 \ $x_3 x_4$	00	01	11	10
0	0	0	-	0
1	1	1	-	1

Intersection

x_2 \ $x_3 x_4$	00	01	11	10
0	0	0	1	0
1	0	0	1	0

Example

Decomposition with intersection

$x_1 x_2$ \ $x_3 x_4$	00	01	11	10
00	0	1	1	0
01	0	0	1	0
11	1	1	1	1
10	0	0	1	0

$x_1 = 0$

x_2 \ $x_3 x_4$	00	01	11	10
0	0	1	1	0
1	0	0	0	0

$x_1 = 1$

x_2 \ $x_3 x_4$	00	01	11	10
0	0	0	0	0
1	1	1	1	1

Intersection

x_2 \ $x_3 x_4$	00	01	11	10
0	0	0	1	0
1	0	0	1	0

The cubes are not split

Example 2

Let $x = x_1$

$x_1 x_2$ \ $x_3 x_4$	00	01	11	10
00	1	0	0	1
01	1	1	0	1
11	0	1	1	1
10	1	0	0	1

$x_1 = 0$

x_2 \ $x_3 x_4$	00	01	11	10
0	-	0	0	-
1	1	-	0	-

$x_1 = 1$

x_2 \ $x_3 x_4$	00	01	11	10
0	-	0	0	-
1	0	-	1	-

x_2 \ $x_3 x_4$	00	01	11	10
0	1	0	0	1
1	0	1	0	-

Intersection

Example 2

Let $x = x_1$

$x_1 x_2$ \ $x_3 x_4$	00	01	11	10
00	1	0	0	1
01	1	1	0	1
11	0	1	1	1
10	1	0	0	1

$x_1 = 0$

x_2 \ $x_3 x_4$	00	01	11	10
0	-	0	0	-
1	1	-	0	-

$x_1 = 1$

x_2 \ $x_3 x_4$	00	01	11	10
0	-	0	0	-
1	0	-	1	-

Intersection

x_2 \ $x_3 x_4$	00	01	11	10
0	1	0	0	1
1	0	1	0	1

P-representation of a completely specified Boolean function f

- Let $$I = f_{x=0} \cap f_{x\neq 0}$$

- A P-representation P(f) (or P-circuit) is:

$$P(f) = x\, f^{\neq} + \bar{x}\, f^{=} + f^{I}$$

where

$$f_{x=0} \setminus I \subseteq f^{=} \subseteq f_{x=0}$$
$$f_{x\neq 0} \setminus I \subseteq f^{\neq} \subseteq f_{x\neq 0}$$
$$\emptyset \subseteq f^{I} \subseteq I$$
$$P(f) = f$$

Minimization of P-circuits using Boolean Relation

- P-circuit minimization:
  - find the sets $f^{=}$, $f^{\neq}$, f^{I} leading to a P-circuit of minimal cost
- Formalized and solved using Boolean relations
- We define a relation R such that
  - the set of all the compatible functions of R corresponds exactly to the set of all possible P-circuits for f
  - an optimal solution of R is an optimal P-circuit for f

Minimization of P-circuits using Boolean Relation

- $f: \{0,1\}^n \rightarrow \{0,1\}$ $R_f : \{0,1\}^{n-1} \rightarrow \{0,1\}^3$
- Input set for R_f: all the variables but the critical signal x_i
- Output set for R_f: triple of functions $f^{=}$, $f^{\neq}$, f^{I} defining a P-circuit for f

$x_1 \ldots x_{i-1} x_{i+1} \ldots x_n$	R_f $(f^{=}, f^{\neq}, f^{I})$
Points in $f_{xi=0} \setminus I$	100
Points in $f_{xi\neq 0} \setminus I$	010
Points in I	{- -1, 11-} = {001, 011, 101, 111, 110}
All other points	000

Minimization of P-circuits using Boolean Relation

Theorem:

P-circuit minimization for f

$\Longleftrightarrow$

minimization of the Boolean relation R_f

Incompletely Specified Functions

$f_{xi=1}$

$f_{xi=0}$

	0	1	-
0	000	010	0-0
1	100	{--1, 11-}	{--1, 1--}
-	-00	{--1, -1-}	---

P-circuit of an incompletely specified Boolean function f

- Let $f = \{f^{on}, f^{dc}\}$, with $f^{on} \cap f^{dc} = \varnothing$;
- Define $I = (f_{x=0}^{on} \cup f_{x=0}^{dc}) \cap (f_{x\neq 0}^{on} \cup f_{x\neq 0}^{dc})$
- A **P-circuit P(f)** is:

$$P(f) = x\, f^{\neq} + \bar{x}\, f^{=} + f^{I}$$

where

$$f_{x=0}^{on} \setminus I \subseteq f^{=} \subseteq f_{x=0}^{on} \cup f_{x=0}^{dc}$$

$$f_{x\neq 0}^{on} \setminus I \subseteq f^{\neq} \subseteq f_{x\neq 0}^{on} \cup f_{x\neq 0}^{dc}$$

$$\varnothing \subseteq f^{I} \subseteq I$$

$$f^{on} \subseteq P(f) \subseteq f^{on} \cup f^{dc}$$

Experimental results

- Linux Intel Core i7, 3.40 GHz CPU, 8GB RAM
- CUDD library for OBDDs for function representation
- BREL (Bañeres, Cortadella, and Kishinevsky, 2009) for the synthesis of Boolean relations
- Multioutput benchmarks have been synthesized minimizing each single output independently from the others

Experimental results

- μ_L and μ_{BDD}:
  - refer to P-circuits synthesized with cost function μ_L that minimizes the number of literals
  - and μ_{BDD} that minimizes the size of the BDDs used for representing the relations
- modeling the P-circuit minimization problem using Boolean relations pays significantly:
  - P-circuit μ_L and P-circuit μ_{BDD} turned out to be more compact than the corresponding P-circuits proposed BCVT2009 in about 92% and 78% of our experiments, respectively

Experimental results

Average gain	P-circuit μ_L Time	P-circuit μ_L Area	P-circuit μ_L Delay	P-circuit μ_{BDD} Time	P-circuit μ_{BDD} Area	P-circuit μ_{BDD} Delay
w.r.t. S-circuit	-383%	37%	29%	95%	30%	25%
w.r.t. P-circuit [4]	-4214%	33%	24%	56%	25%	20%
w.r.t. SOP	-39412%	26%	19%	-304%	18%	14%

TABLE II

AVERAGE GAIN OF P-CIRCUITS BASED ON BOOLEAN RELATIONS

	P-circuit μ_L	P-circuit μ_{BDD}
w.r.t. S-circuit	65%	61%
w.r.t. P-circuit [4]	13%	13%
w.r.t. SOP	44%	62%

TABLE III

COMPARISON OF POWER DISSIPATION

Conclusions

- Boolean relations can be useful for modeling Boolean hard optimization problems
- Boolean relations have been successfully used in logic synthesis
- Future work:
  - investigating the use of Boolean relations in other algorithmic contexts (i.e., data mining)
  - trade-off quality of results vs. scalability

Thanks

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